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RB-0762

Second Year B. Sc. (Computer Science) Examination

April / May – 2010

Mathematics : Paper - III

(Old Course)

Time : **3** Hours] [Total Marks : **105**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :

☛ **S.Y. B.Sc. (COMPUTER SCIENCE)**

Name of the Subject :

☛ **MATHEMATICS - 3 (OLD)**

☛ Subject Code No. : **0 7 6 2** ☛ Section No. (1, 2,.....) : **NIL**

Seat No. :

Student's Signature

(2) All questions are compulsory.

(3) Figures to the right indicate full marks.

1 Answer the following questions : **15**

(1) State Euler's theorem for a homogeneous functions.

(2) Write the expression for Maclaurin's expansion of a bivariate functions.

(3) Evaluate $\int_0^3 \int_0^4 (x-y)\,dx\,dy$

(4) Explain why Beta and gamma integrals are improper.

(5) Find f_{xy} for $f(x, y) = \log\left(x^2 + y^2\right)$.

(6) Write the properties of Jacobian.

(7) Solve $x^3\frac{d^3y}{dx^3}-x^2\frac{d^2y}{dx^2}+2x\frac{dy}{dx}-2y=0$

(8) Find the particular integral of $\frac{d^3y}{dx^3}-3\frac{d^2y}{dx^2}+3\frac{dy}{dx}-y=e^x$

(9) Write the general form of Legendre's linear differential equation.

(10) Find the value of $\beta\left(\frac{1}{2},\frac{1}{2}\right)$.

2 (a) Define continuity of a bivariate function. Discuss the continuity of **6**

$$f(x,y)=\frac{x^2+y^2}{x^2y^2+(x-y)^2} \quad ; x\neq y$$
$$=2 \quad ; x=0,\ y=0$$

at the point $(0,0)$

(b) If $z=f(u,v)$ and $u=e^x\cos y,\ \ v=e^x\sin y$ then prove **6**

that $\frac{\partial^2z}{\partial x^2}+\frac{\partial^2z}{\partial y^2}=\left(u^2+v^2\right)\left(\frac{\partial^2z}{\partial u^2}+\frac{\partial^2z}{\partial v^2}\right)$

(c) If $u=\tan^{-1}\frac{x^3+y^3}{x-y}$ then prove that **6**

$$x^2\frac{\partial^2u}{\partial x^2}+2xy\frac{\partial^2y}{\partial x\partial y}+y^2\frac{\partial^2y}{\partial y^2}=\sin 4u-\sin 2u$$

OR

2 (a) If $f(x, y) = x^2 \tan^{-1}\frac{y}{x} - \tan^{-1}\frac{x}{y}$ $\quad (x, y) \neq (0,0)$ **6**

$\quad = 0 \quad (x, y) = (0,0)$

then find $f_{xy}(0,0)$, $f_{yx}(0,0)$, $f_{xx}(0,0)$, $f_{yy}(0,0)$.

(b) Verify Euler's theorem for the function **6**

$$f(x, y) = \tan^{-1}\frac{\sqrt{x} - \sqrt{y}}{\sqrt{x} + \sqrt{y}} \quad (x, y) \neq (0,0)$$

(c) Find the limit of $f(x, y) = x^2 + 2y$ when $(x, y) \to (1, 3)$ **6**

using its definition.

3 (a) Expand $e^x \log(1+y)$ in powers of x and y upto **6**

terms of third degree.

(b) Find the maximum and minimum values of **6**

$x^3 y^2 (1 - x - y)$.

(c) If $x\sqrt{1-y^2} + y\sqrt{1-x^2} = a$ then show that **6**

$$\frac{d^2y}{dx^2} = \frac{-a}{\left(1-x^2\right)^{3/2}}$$

OR

3 (a) If $x+y+z=u$, $y+z=v$, $z=uvw$ then obtain $J(x, y, z)$. **6**

(b) Prove that **6**

$$\sin x \cdot \sin y = xy - \frac{1}{6}\left\{\left(x^3+2xy^2\right)\cos\theta_x \sin\theta_y + \left(y^3+3x^2y\right)\sin\theta_x \cdot \cos\theta_y\right\}$$

where $0<\theta<1$

(c) If $z(x+y)=x^2+y^2$ then prove that **6**

$$\left(\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}\right)^2 = 4\left(1-\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}\right)$$

4 (a) Evaluate the integral after changing the order of integration **6**

$$\int_0^1 \int_y^{2-y} \left(x^2+y^2\right) dx dy$$

(b) Prove that **6**

$$\Gamma n = \int_0^1 \left(\log\frac{1}{y}\right)^{n-1} dy$$

(c) Evaluate : **6**

$$\int_0^2 \frac{x^2}{\sqrt{2-x}} dx$$

OR

4 (a) Evaluate $\iint_S xy\,dx\,dy$ where S is the region bounded by the lines $y=0, x=2$ and $x^2=4y$. **6**

(b) Show that $\int_0^2 x\left(8-x^3\right)^{1/3} dx = \frac{16\pi}{9\sqrt{3}}$ **6**

(c) Prove that $\overline{|1+n} = n\overline{|n}$ **6**

5 (a) Discuss the method to find the particular integral of the n^{th} order linear differential equation with constant coefficient when $X = e^{ax}v$ where a is any constant and v is any function of x. **6**

(b) Solve : $\frac{d^2y}{dx^2} - y = x^2 \cos x$ **6**

(c) Solve : $\frac{d^3y}{dx^3} + y = e^{x/2} \cdot \sin\frac{\sqrt{3}}{2}x$ **6**

OR

5 (a) In usual notation prove that **6**

$$\frac{1}{f(D)} xv = \left\{x - \frac{1}{f(D)} \cdot f'(D)\right\} \frac{1}{f(D)} \cdot V$$

where V is a function of x.

(b) Solve $\frac{d^3y}{dx^3} - y = e^x \cos x$ **6**

(c) Solve : $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 13y = 8e^{3x} \sin 2x$ **6**

6 (a) Solve : $x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right)$ **6**

(b) Solve the following equation using Gauss-Jordan method : **6**

$2x - 3y + 4z = 7$

$5x - 2y + 2z = 7$

$6x - 3y + 10z = 23$

(c) Solve the following equation using Gauss-Seidel method **6**

$10x + y + z = 12$

$2x + 10y + z = 13$

$2x + 2y + 10z = 14$

OR

6 (a) Solve : **6**

$$(3x+2)^2 \frac{d^2y}{dx^2} + 3(3x+2)\frac{dy}{dx} - 36y = 3x^2 + 4x + 1$$

(b) Solve the following equation using Gauss-Elimination method : **6**

$9x - 2y - z = 17$

$4x + 5y - 2z = -9$

$x - 3y - 5z = 4$

(c) Solve the following equation using Gauss Jordan method : **6**

$5x + 2y + z = 12$

$x + 4y + 2z = 15$

$x + 2y + 5z = 20$
